Oscillating Magnet WEC

Dynamics / Spring Selection

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March 2026

Contents

Part I
System Dynamics

1 System Description

A symmetric rigid array of permanent magnets is sandwiched between two parallel planar surfaces inside a flat circular housing. The magnets roll on roller ball bearings such that friction is negligible. Coils are wound directly into both the top and bottom surfaces, so every magnet induces EMF simultaneously in two coil layers. Linear springs connect the array frame to the housing wall, providing the sole restoring force (compared to previously having a curved bowl). The housing is mounted so that the flat face parallel to the ocean surface. As ocean waves pass, the entire system tilts and the magnet array slides relative to the housing.

The mounting height h of the housing above the buoy's tilt pivot is adjustable. The boat rocking creates an additional inertial forcing correction that depends on the frequency and mounting height.

1.1 Key Simplifications Relative to Curved-Bowl Designs

1. **No curvature** ($R_s \to \infty$): the geometric gravity restoring term g/R_s vanishes exactly. Springs are the only restoring mechanism.

2. **Translational motion**: the degree of freedom is displacement $u(t)$ in the horizontal plane, not a rotation angle. The problem is 2D in the sliding plane but reduces to 1D for a single-axis wave.

3. **No null point** (at $h = 0$): the forcing amplitude $F_0 = g\alpha_0$ is independent of wave frequency, so there is no cancellation frequency. A null point reappears only if $h \neq 0$, but both terms are then additive (not opposing), so the null point cannot occur for $h > 0$ either.

4. **Double-sided induction**: coils on both surfaces double the effective load damping c_L relative to a single-sided arrangement.

1.2 Parameter Definitions

Symbol	Definition	Notes
$u(t)$	Displacement of array CoM relative to housing centre	Scalar (1D)
$\alpha(t)$	Tilt angle of buoy/housing	Prescribed, small
h	Height of housing above buoy tilt pivot	Adjustable via buoyancy
M_{array}	Total mass of magnet array and frame	
k	Combined spring stiffness (all springs in parallel)	
c_i	Internal (friction) damping coefficient	
c_L	Load (harvester) damping coefficient	From both coil surfaces
c_T	Total damping: $c_i + c_L$	
g	Gravitational acceleration	$9.81\,\mathrm{m/s^2}$
ω_n	Undamped natural frequency	$\sqrt{k/M_{array}}$
Ω	Wave excitation frequency	$2\pi/T_{wave}$
α_0	Buoy tilt amplitude	From wave environment
F_0	Forcing amplitude (per unit mass)	$\alpha_0(g + h\Omega^2)$
$\zeta_i, \zeta_L, \zeta_T$	Internal, load, total damping ratios	

2 Coordinate System and Kinematics

2.1 Lab Frame

Origin fixed to the undisturbed buoy tilt pivot. X-axis horizontal (positive in the tilt direction), Y-axis vertical upward, Z-axis out of page. The housing tilts with the buoy at angle $\alpha(t)$ about the pivot.

2.2 Housing Frame

A non-inertial frame fixed to the tilting housing. In this frame the effective gravity vector rotates, driving the array to slide. For small α:

$$g_{eff,x} \approx g \sin\alpha \approx g\alpha \tag{1}$$

2.3 Position of Array CoM

The housing center is at height h above the pivot. In the previously defined frame:

$$\vec{r}_{housing} = \begin{pmatrix} h\sin\alpha \\ h\cos\alpha \end{pmatrix} \approx \begin{pmatrix} h\alpha \\ h \end{pmatrix} \tag{2}$$

The array CoM, displaced by u along the housing x-direction using small angle approximation:

$$X_b = h\sin\alpha + u\cos\alpha \approx h\alpha + u \tag{3}$$

$$Y_b = h\cos\alpha - u\sin\alpha \approx h - u\alpha \tag{4}$$

At $\alpha = u = 0$, $(X_b, Y_b) = (0, h)$, the array is at rest directly above pivot.

2.4 Array Velocity

Differentiating to first order in small quantities:

$$\dot{X}_b = h\dot{\alpha} + \dot{u} \tag{5}$$

$$\dot{Y}_b \approx 0 \quad \text{(second-order, negligible)} \tag{6}$$

Velocity squared approximates from above:

$$v^2 = X_b^2 + Y_b^2 \tag{7}$$

$$v^2 \approx (h\dot{\alpha} + \dot{u})^2 \tag{8}$$

3 Lagrangian Derivation

3.1 Kinetic Energy

Since the array slides without rolling, kinetic energy is purely translational:

$$T = \frac{1}{2}M_{array}v^2 = \frac{1}{2}M_{array}(h\dot{\alpha} + \dot{u})^2 \tag{9}$$

Expanding:

$$T = \frac{1}{2}M_{array}\left(h^2\dot{\alpha}^2 + 2h\dot{\alpha}\dot{u} + \dot{u}^2\right) \tag{10}$$

The $h^2\dot{\alpha}^2$ term depends only on the prescribed motion and does not affect the equation of motion for u. The dynamically relevant terms are:

$$T_{relevant} = \frac{1}{2}M_{array}\dot{u}^2 + M_{array}h\dot{\alpha}\dot{u} \tag{11}$$

3.2 Potential Energy

Gravitational:

$$U_{grav} = M_{array}gY_b \approx M_{array}g(h - u\alpha) \tag{12}$$

The constant $M_{array}gh$ does not affect the dynamics. The relevant term is:

$$U_{grav,relevant} = -M_{array}gu\alpha \tag{13}$$

Spring:

$$U_{spring} = \frac{1}{2}ku^2 \tag{14}$$

Total relevant potential:

$$U = \frac{1}{2}ku^2 - M_{array}gu\alpha \tag{15}$$

3.3 Lagrangian

$$L = T - U = \frac{1}{2}M_{array}\dot{u}^2 + M_{array}h\dot{\alpha}\dot{u} - \frac{1}{2}ku^2 + M_{array}gu\alpha \tag{16}$$

3.4 Euler-Lagrange Equation for u

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{u}}\right) - \frac{\partial L}{\partial u} = Q_u \tag{17}$$

$\partial L/\partial \dot{u}$:

$$\frac{\partial L}{\partial \dot{u}} = M_{array}\dot{u} + M_{array}h\dot{\alpha} \tag{18}$$

$d/dt\,(\partial L/\partial \dot{u})$:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{u}}\right) = M_{array}\ddot{u} + M_{array}h\ddot{\alpha} \tag{19}$$

$\partial L/\partial u$:

$$\frac{\partial L}{\partial u} = -ku + M_{array}g\alpha \tag{20}$$

Non-Conservative Generalized Forces (Damping): To account for energy dissipation, we define a generalized force Q_u that encompasses both mechanical losses and electrical power extraction. The internal mechanical losses are modeled as a continuous, low-resistance linear damping term.

- **Internal Damping (c_i):** Friction from contact patches of the ball bearing and additional friction within assembly.
- **Load Damping (c_L):** Damping from magnetic braking due to resstive force that comes from induced back-EMF.

Summing these effects, the non-conservative generalized force is defined as:

$$Q_u = -(c_i + c_L)\dot{u} \tag{21}$$

Assembling the equation of motion:

$$M_{array}\ddot{u} + M_{array}h\ddot{\alpha} + ku - M_{array}g\alpha = -(c_i + c_L)\dot{u} \tag{22}$$

Rearranging into the standard forced-oscillator form:

$$\boxed{M_{array}\ddot{u} + (c_i + c_L)\dot{u} + ku = M_{array}g\alpha(t) - M_{array}h\ddot{\alpha}(t)} \tag{23}$$

The right-hand side contains two distinct forcing mechanisms:

1. **Gravity forcing** $M_{array}g\alpha$: the tilt of the housing causes gravity to have a component along the sliding direction. Dominant at low frequencies; independent of wave frequency.
2. **Inertial forcing** $-M_{array}h\ddot{\alpha}$: the housing centre accelerates horizontally as the buoy tilts. Proportional to h and to Ω^2; vanishes at $h = 0$. For $h > 0$ both terms are *additive* (not opposing), so there is no null point.

4 Natural Frequency

Leaving the system along by setting $\alpha = 0$ and $c = 0$:

$$M_{array}\ddot{u} + ku = 0 \quad \Longrightarrow \quad \boxed{\omega_n = \sqrt{\frac{k}{M_{array}}}} \tag{24}$$

This contains no geometric parameters, no gravitational correction, and no rolling factor. The required spring stiffness for a target ω_n is:

$$\boxed{k = M_{array}\,\omega_n^2} \tag{25}$$

5 Damping and Resonate Amplitude

5.1 Excitation

Assuming the wave prescribe a sinusoidal acceleration of $\alpha(t) = \alpha_0 \sin(\Omega t)$, $\ddot{\alpha} = -\alpha_0 \Omega^2 \sin(\Omega t)$.

5.2 Forcing Amplitude

Substituting into Equation (23) and dividing by M_{array}:

$$\ddot{u} + 2\zeta_T \omega_n \dot{u} + \omega_n^2 u = F_0 \sin(\Omega t) \tag{26}$$

where:

$$\boxed{F_0 = \alpha_0(g + h\Omega^2)} \tag{27}$$

5.3 Damping Ratio

The damping ratio combines the internal and load components to tell whether the magnets will move back and forth freely (underdamped - target), immediately return to origin (critically damped), or slowly return (overdamped).

$$\boxed{\zeta_T = \frac{c_i + c_L}{2M_{array}\omega_n}}, \qquad \zeta_i = \frac{c_i}{2M_{array}\omega_n}, \qquad \zeta_L = \frac{c_L}{2M_{array}\omega_n} \tag{28}$$

6 Frequency Response Derivation

The sliding array is modeled as a forced, damped second-order system:

$$m\ddot{u} + c\dot{u} + ku = F_0 \cos(\Omega t) \tag{29}$$

Define the natural frequency and damping ratio:

$$\omega_n = \sqrt{\frac{k}{m}}, \qquad \zeta_T = \frac{c}{2m\omega_n} \tag{30}$$

Dividing the equation of motion by m:

$$\ddot{u} + 2\zeta_T \omega_n \dot{u} + \omega_n^2 u = \frac{F_0}{m} \cos(\Omega t) \tag{31}$$

6.1 Steady-State Harmonic Response

Source 1: https://phys.libretexts.org/Bookshelves/Classical$_{M}echanics/Classical_{M}echanics$**Source 2** : $https : //www.acs.psu.edu/drussell/Demos/Resonance - Regions/Resonance.html$

Assume a steady-state sinusoidal response of the form:

$$u(t) = U\cos(\Omega t - \phi) \tag{32}$$

Using phasor representation, the transfer function between displacement and forcing is:

$$U(\Omega) = \frac{F_0/m}{\omega_n^2 - \Omega^2 + i(2\zeta_T\omega_n\Omega)} \tag{33}$$

Taking the magnitude:

$$|U(\Omega)| = \frac{F_0/m}{\sqrt{(\omega_n^2 - \Omega^2)^2 + (2\zeta_T\omega_n\Omega)^2}} \tag{34}$$

Factoring out ω_n^2:

$$|U(\Omega)| = \frac{F_0}{m\omega_n^2} \cdot \frac{1}{\sqrt{\left(1 - \left(\frac{\Omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta_T\frac{\Omega}{\omega_n}\right)^2}} \tag{35}$$

Since $k = m\omega_n^2$, this simplifies to:

$$|U(\Omega)| = \frac{F_0/\omega_n^2}{\sqrt{\left(1 - \left(\frac{\Omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta_T\frac{\Omega}{\omega_n}\right)^2}} \tag{36}$$

6.2 Resonant Amplitude ($|U|_{res}$)

At resonance ($\Omega = \omega_n$), the stiffness and inertial terms cancel:

$$1 - \left(\frac{\Omega}{\omega_n}\right)^2 = 0 \tag{37}$$

Thus the amplitude reduces to:

$$|U|_{res} = \frac{F_0/\omega_n^2}{2\zeta_T} = \frac{F_0}{2\zeta_T\omega_n^2} \tag{38}$$

Substituting the forcing model:

$$F_0 = \alpha_0(g + h\omega_n^2) \tag{39}$$

gives:

$$|U|_{res} = \frac{\alpha_0(g + h\omega_n^2)}{2\zeta_T\omega_n^2} \tag{40}$$

6.3 Physical Extension Limit

To prevent mechanical impact, the resonant amplitude must be strictly bounded by the housing clearance u_{max} (inner housing radius minus array outer radius):

$$|U|_{res} \leq u_{max} \tag{41}$$

Part II

Spring Selection

7 Specification from the Equation of Motion

The dynamics derivation (Part I) gets:

$$k = M_{array}\,\omega_n^2 \tag{25}$$

7.1 Working Numbers

Parameter	**Value**	**Source**
M_{array} (19 magnets + PLA frame)	≈ 0.57 kg	Mass calculation
Target f_n	≥ 0.67 Hz	Static offset constraint
Required k_{total}	≥ 10 N/m	$k = M\omega_n^2$ at 0.67 Hz
Number of springs n_s	3 (at 120°)	Symmetry, isotropy
Max displacement u_{max}	~150 mm	Housing clearance

7.2 Stiffness Table by Target Frequency

The corresponding spreadsheet can be found at: Stiffness Table Spreadsheet.

8 Implementation

When researching implementation options we settled on either:

1. **Option 1: Elastic Cords.**
2. **Option 2: 302 Stainless Steel Springs**

8.1 Option 1: Elastic Cord

Thin silicone or latex cord in tension acts as an easily adjustable linear spring. The stiffness is defined by:

$$k_{cord} = \frac{E\,A}{L_0} \tag{42}$$

For a 0.5 mm diameter silicone cord ($E \approx 1.5$ MPa), an 88 mm length provides the target $k \approx 3.35$ N/m.

8.2 Option 2: 302 Stainless Steel Springs

Source: https://roymech.org/Useful$_{T}ables/Springs/Springs_{h}elical.htmlThestiffnessofahelicalspri$ $\frac{G\,d_w^4}{8\,D_c^3\,N_t}$(43) Using standard 0.20 mm wire (d_w) and a 1.60 mm mean coil diameter (D_c), the required number of active turns (N_t) scales inversely with target stiffness.

Preload Requirement: To ensure linear behavior and prevent the array from slacking under sustained tilt ($\bar{\alpha}$), the installed springs must have a minimum tension preload:

$$F_{preload} > \frac{M_{array}\, g\, \bar{\alpha}}{n_s} \tag{44}$$