Oscillating Magnet Wave Energy Converter

Simulation Framework

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Contents

1 Introduction and Motivation

The wave energy converter uses oscillating linear generators where cylindrical magnts slide freely along an inclined rail under gravity (and in full deployment, wave forcing). As the sled passes through a stationary coil array, time-varying magnetic flux induces EMF in each coil, driving current through an electrical load.

The general model for this system follows the following lumped parameter model:

$$V_{\text{oc}} = N B_{\text{eff}} \ell_{\text{eff}} \, v, \tag{1}$$

where N is turn count, B_{eff} an effective flux density, and ℓ_{eff} an effective conductor length. For our specific use case, this approximation doesn't do a good of accounting for reality because it:

- Assumes a spatially uniform field, ignoring the strongly non-uniform distribution of a short magnet array passing through finite coils.
- Cannot represent the sign change of dB_z/du as a magnet pole cener crosses a coil center, which is responsible for the characteristic double-peak in the transduction coefficient $K(u)$.
- Treats all coils as contributing equally, whereas in reality coils far from the sled contribute negligible flux at any given instant.
- Cannot capture the nonlinear parallel-network behaviour when coils with different induced voltages share a common load.

To approach this problem, the code follows an approach that first:

- Computes the spatial magnetic field via a double numerical Biot–Savart integral over the magnet surface (Section 3).
- Evaluating the flux linkage through the annular coil geometry (Section 4).
- Translate the flux linkage with the diode rectification and circuitry (Section 5).
- Integrating the fully coupled equations of motion with an ODE solver (Section 6).

2 System Configuration

2.1 Coordinate Systems

Two coordinate systems are used simultaneously.

Rail (generalised) coordinate. The scalar u denotes the displacement of the magnet sled along the rail from the track center:

$$u \in \left[-\frac{L}{2},\, +\frac{L}{2}\right], \qquad L = 0.4\,\text{m}. \tag{2}$$

$u = -L/2$ is the top of the rail (start); $u = +L/2$ is the bottom (end). The rail is inclined at $\theta = 30^\circ$ from horizontal. The sled velocity along the rail is:

$$v = \frac{du}{dt}. \tag{3}$$

Magnet-local cylindrical coordinate. For the field calculation of a single magnet, a cylindrical frame (r, z) is attached to that magnet's centre of mass:

- $z = 0$ at the magnet midplane; z is positive toward the upper coil bank.
- $r \geq 0$ is the radial distance from the magnet axis of symmetry.

The lab-frame coordinate x along the track (used for the field map) is related to the magnet-local frame by $x_{\text{rel}} = x - x_m$, where x_m is the current position of magnet m along the rail.

2.2 Magnet Array

$N_{\text{mag}} = 4$ identical cylindrical NdFeB disc magnets are mounted on the sled in an alternating N–S–N–S polarity pattern. Alternating polarity maximizes $|dK/du|$ and therefore the peak induced voltage for a given velocity.

Polarity.

$$p(m) = \begin{cases} +1 & m = 0, 2, 4, \ldots \quad \text{(N pole facing } +z) \\ -1 & m = 1, 3, 5, \ldots \quad \text{(S pole facing } +z) \end{cases} \tag{4}$$

Magnet centre positions. Each magnet is offset from the sled centre by an integer multiple of the magnet pitch s_m:

$$x_m(u) = u + \left(m - \frac{N_{\text{mag}} - 1}{2}\right) s_m, \qquad m = 0, 1, \ldots, N_{\text{mag}} - 1. \tag{5}$$

Key magnet parameters.

Symbol	Value	Unit	Description
B_r	1.45	T	Remanent flux density (NdFeB)
R_m	12.5	mm	Magnet outer radius
d_m	25	mm	Magnet axial thickness
s_m	30	mm	Centre-to-centre magnet pitch
M_{single}	92	g	Mass per magnet
M	418	g	Total sled + magnet mass

2.3 Coil Array

The coil array is stationary and surrounds the magnet channel. $N_{\text{cps}} = 5$ coils per side are arranged in two banks (top and bottom), giving $N_c = 10$ coils total. Both banks share the same axial positions but sit at $z = +z_{\text{dist}}$ (top) and $z = -z_{\text{dist}}$ (bottom) relative to the magnet midplane.

Coil axial positions.

$$x_{c,i} = \left(i - \frac{N_{\text{cps}} - 1}{2}\right) s_c, \qquad i = 0, 1, \ldots, N_{\text{cps}} - 1, \tag{6}$$

Coil-plane offset. The coils are separated from the magnet body by the air gap plus half the coil depth:

$$z_{\text{dist}} = \frac{d_m}{2} + g_{\text{air}} + \frac{d_c}{2} = 12.5 + 1.2 + 6.25 = 19.95\,\text{mm}. \tag{7}$$

Key coil parameters.

Symbol	Value	Unit	Description
R_{in}	8	mm	Inner winding radius
R_{out}	21	mm	Outer winding radius
d_c	12.5	mm	Axial coil depth
s_c	30	mm	Coil pitch
N	1200	—	Turns per coil
R_{int}	36	Ω	Internal resistance per coil
z_{dist}	19.95	mm	Coil-plane offset from magnet midline

3 Magnetic Field Model

3.1 Equivalent Surface Current Representation

A uniformly magnetised cylinder with remanence B_r and relative permeability $\mu_r \approx 1$ (hard magnet) is rigorously equivalent to a surface current sheet on its curved face. The volumetric magnetisation is:

$$\mathbf{M} = \frac{B_r}{\mu_0}\,\hat{z}, \tag{8}$$

and the equivalent surface current density on the lateral face ($r = R_m$, $-d_m/2 \leq z \leq d_m/2$) is:

$$\mathbf{K}_s = \mathbf{M} \times \hat{n} = \frac{B_r}{\mu_0}\,\hat{\phi}, \tag{9}$$

where $\hat{n} = \hat{r}$ is the outward normal and $\hat{\phi}$ is the azimuthal unit vector. This sheet current flows in circular loops, identical in form to a solenoid of finite length. The total surface current per unit length is $K_s = B_r/\mu_0 \approx 1.154 \times 10^6$ A/m.

3.2 Biot–Savart Law

The magnetic field produced by an infinitesimal current element $I\,d\boldsymbol{\ell}$ at position $\mathbf{r}'$, evaluated at field point $\mathbf{r}$, is:

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\,\frac{I\,d\boldsymbol{\ell} \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}. \tag{10}$$

For the surface current sheet, the integration is carried out over both the azimuthal angle $\theta \in [0, 2\pi]$ and the axial source position $z_q \in [-d_m/2, +d_m/2]$.

3.3 Axial Field Component B_z

Substituting the surface current into Eq. (10) and projecting onto $\hat{z}$, the axial field at evaluation point (r_p, z_p) due to a single magnet centred at the origin is:

$$B_z(r_p, z_p) = -\frac{B_r}{4\pi} \int_{-d_m/2}^{d_m/2} \int_0^{2\pi} \frac{R_m\,(r_p \cos\theta - R_m)}{\left[(r_p - R_m \cos\theta)^2 + (R_m \sin\theta)^2 + (z_p - z_q)^2\right]^{3/2}}\, d\theta\, dz_q. \tag{11}$$

Derivation of the numerator factor. At source point $(R_m \cos\theta,\, R_m \sin\theta,\, z_q)$, the surface current element is $d\boldsymbol{\ell} = R_m\, d\theta\, \hat{\phi} = R_m\, d\theta\, (-\sin\theta, \cos\theta, 0)$. The displacement vector to field point $(r_p, 0, z_p)$ in Cartesian coordinates is:

$$\mathbf{R} = (r_p - R_m \cos\theta,\ -R_m \sin\theta,\ z_p - z_q). \tag{12}$$

The cross product $d\boldsymbol{\ell} \times \mathbf{R}$ has z-component:

$$(d\boldsymbol{\ell} \times \mathbf{R})_z = R_m\, d\theta \left(-\sin\theta \cdot (-R_m \sin\theta) + \cos\theta \cdot (r_p - R_m \cos\theta)\right) = R_m\, d\theta\, (r_p \cos\theta - R_m). \tag{13}$$

The denominator $|\mathbf{R}|^3$ becomes the expression under the integral in Eq. (11), and the prefactor $K_s = B_r/\mu_0$ with the $\mu_0/4\pi$ from Biot–Savart yields the $B_r/4\pi$ factor.

Numerical evaluation. The integral is evaluated using the composite trapezoidal rule over a $(n_\theta \times n_z) = (60 \times 20)$ grid:

$$B_z \approx -\frac{B_r}{4\pi} \sum_{j=0}^{n_z-1} \sum_{i=0}^{n_\theta-1} w_i^{(\theta)} w_j^{(z)}\, \frac{R_m(r_p \cos\theta_i - R_m)}{D_{ij}^{3/2}}, \tag{14}$$

where $D_{ij} = (r_p - R_m \cos\theta_i)^2 + (R_m \sin\theta_i)^2 + (z_p - z_{q,j})^2 + \varepsilon$, with regularisation $\varepsilon = 10^{-9}$ to prevent division by zero, and $w_i^{(\cdot)}$ are the trapezoidal weights.

3.4 Radial Field Component B_r

The radial component B_r is required for the streamplot field-line visualisation. Rather than repeating the full Biot–Savart derivation for B_r, it is obtained from B_z via Maxwell's divergence-free condition $\nabla \cdot \mathbf{B} = 0$. In cylindrical coordinates:

$$\frac{1}{r}\frac{\partial(rB_r)}{\partial r} + \frac{\partial B_z}{\partial z} = 0. \tag{15}$$

Integrating from $r = 0$ (where $B_r = 0$ by symmetry) and approximating the radial variation as linear for small r:

$$B_r(r_p, z_p) \approx -\frac{r_p}{2} \frac{\partial B_z}{\partial z}\bigg|_{r_p, z_p}, \tag{16}$$

with the z-derivative evaluated by central finite difference:

$$\frac{\partial B_z}{\partial z}\bigg|_{z_p} \approx \frac{B_z(r_p, z_p + \delta z) - B_z(r_p, z_p - \delta z)}{2\delta z}, \qquad \delta z = 10^{-4}\,\text{m}. \tag{17}$$

This approximation is exact on axis and remains accurate for $r \ll R_m$.

3.5 Field Superposition

The total field of the magnet array at lab-frame position (x, z) is obtained by summing contributions from all magnets with their polarity factors:

$$B_z^{\text{total}}(x, z; u) = \sum_{m=0}^{N_{\text{mag}}-1} p(m)\; B_z(|x - x_m(u)|,\; z)\,, \tag{18}$$

$$B_x^{\text{total}}(x, z; u) = \sum_{m=0}^{N_{\text{mag}}-1} p(m)\; B_r(|x - x_m(u)|,\; z) \cdot \text{sgn}(x - x_m(u)), \tag{19}$$

where $|x - x_m|$ is the radial cylindrical distance to magnet m's axis, and the sgn factor in Eq. (19) projects the cylindrical radial component B_r onto the lab-frame x-direction.

4 Flux Linkage and Electromagnetic Coupling

4.1 Flux Linkage Through an Annular Coil

The flux threading a single turn of wire lying in the plane $z = z_{\text{dist}}$ and spanning radii R_{in} to R_{out} is:

$$\phi_{\text{turn}}(u) = \int_{R_{\text{in}}}^{R_{\text{out}}} B_z(r,\; z_{\text{dist}};\; u - x_{c,i})\; 2\pi r\, dr, \tag{20}$$

where $u - x_{c,i}$ is the position of the sled centre relative to coil i. For a coil with N turns, the total flux linkage is:

$$\Phi_i(u) = N \int_{R_{\text{in}}}^{R_{\text{out}}} B_z^{\text{total}}(r,\; z_{\text{dist}};\; u - x_{c,i})\;\; 2\pi r\, dr, \tag{21}$$

where B_z^{total} is the superposed field from the full magnet array (Eq. (18)). The integral is evaluated numerically with the trapezoidal rule over $n_r = 12$ sample points in $[R_{\text{in}}, R_{\text{out}}]$.

Physical interpretation. $\Phi_i(u)$ varies, increasing as the leading magnet approaches, peaks near coil centre, and reverses sign as the next opposite-polarity magnet arrives. The resulting double-peak shape of $d\Phi/du$ is the origin of the AC character of the induced EMF.

4.2 Transduction Coefficient

The transduction coefficient K_i is the rate of change of flux linkage with sled displacement:

$$K_i(u) = \frac{d\Phi_i}{du} \qquad [\text{V} \cdot \text{s/m} \equiv \text{N/A}]\,. \tag{22}$$

The dual unit ($\text{V} \cdot \text{s/m}$ and N/A) reflects the *electromechanical reciprocity* of the transducer: the same coefficient governs both electrical generation (EMF per unit velocity) and mechanical loading (force per unit current). This is a consequence of energy conservation and is exact for any linear transducer.

Numerical computation. K_i is approximated via a one-sided finite difference with step $\Delta u = 5 \times 10^{-4}$ m:

$$K_i(u) \approx \frac{\Phi_i(u + \Delta u) - \Phi_i(u)}{\Delta u}. \tag{23}$$

This is evaluated on a uniform look-up grid of 60 points spanning $[-L/2, +L/2]$ and then interpolated at arbitrary u using a cubic spline, giving a smooth, fast function for use inside the ODE integrator.

4.3 Induced EMF via Faraday's Law

By Faraday's law of induction, the open-circuit EMF in coil i is the time derivative of its flux linkage. Applying the chain rule:

$$V_i(t) = -\frac{d\Phi_i}{dt} = -\frac{d\Phi_i}{du}\frac{du}{dt} = K_i(u)\, v, \tag{24}$$

where the sign convention is absorbed into the polarity of K_i. In vector form for all N_c coils simultaneously:

$$\mathbf{V}(t) = \mathbf{K}(u)\, v, \qquad \mathbf{V}, \mathbf{K} \in \mathbb{R}^{N_c}. \tag{25}$$

5 Electrical Network

5.1 Circuit Model

Each coil is modelled as a Thévenin source: an EMF V_i in series with its DC winding resistance $R_{\text{int}} = 52\,\Omega$. Full-wave rectification is represented by a pair of ideal diodes with a fixed forward voltage drop $V_{\text{drop}} = 0.6$ V, giving an effective voltage:

$$V_{i,\text{eff}} = \max(|V_i| - V_{\text{drop}},\ 0)\,. \tag{26}$$

A coil is *active* if $V_{i,\text{eff}} > 0$, i.e. if its induced EMF exceeds the diode threshold. All active coils are connected in parallel across the load resistance R_L.

5.2 Kirchhoff's Current Law at the Load Node

Let N_{active} be the number of active coils at a given instant. Applying KCL at the common bus connecting all coils to the load:

$$\sum_{i=1}^{N_{\text{active}}} \frac{V_{i,\text{eff}} - V_L}{R_{\text{int}}} = \frac{V_L}{R_L}, \tag{27}$$

where $V_L \geq 0$ is the load voltage. Rearranging:

$$V_L = \frac{\displaystyle\sum_{i=1}^{N_{\text{active}}} V_{i,\text{eff}} \,/\, R_{\text{int}}}{1/R_L + N_{\text{active}}/R_{\text{int}}}. \tag{28}$$

Self-consistent solution algorithm. Equation (28) contains an implicit dependence on N_{active}, since a coil only contributes if its terminal voltage $V_{i,\text{eff}}$ exceeds V_L. The correct N_{active} is found iteratively:

1. Sort coils in descending order of $V_{i,\text{eff}}$.
2. For $N = 1, 2, \ldots, N_c$, compute trial $V_L^{(N)}$ from Eq. (28) using the top N coils.
3. Accept $N_{\text{active}} = N$ when $V_L^{(N)} \geq V_{(N+1),\text{eff}}$ (the $(N+1)$-th coil would not contribute because V_L is already at or above its effective voltage).

5.3 Per-Coil Currents

The current drawn from each active coil, including the correct sign to reflect the instantaneous polarity of the induced EMF, is:

$$I_i = \frac{\max(V_{i,\text{eff}} - V_L,\ 0)}{R_{\text{int}}} \cdot \text{sgn}(V_i). \tag{29}$$

5.4 Load Power

The instantaneous electrical power delivered to the resistive load is:

$$P_L(t) = \frac{V_L(t)^2}{R_L}. \tag{30}$$

The time-averaged power over the full stroke is:

$$\langle P_L \rangle = \frac{1}{T} \int_0^T P_L(t)\, dt, \tag{31}$$

which is the primary figure of merit for generator optimisation.

Maximum power transfer. For a single coil with no diode drop, the load voltage that maximises P_L is $R_L = R_{\text{int}}$, giving the classic matched-load result:

$$P_{L,\max} = \frac{V_{\text{oc}}^2}{4R_{\text{int}}}. \tag{32}$$

With multiple coils in parallel, the optimal load satisfies $R_L = R_{\text{int}}/N_{\text{active}}$, which shifts with the number of contributing coils as the sled moves.

6 Electromechanical Dynamics

6.1 Force Balance

Three forces act on the sled along the rail direction:

Gravitational driving force.

$$F_g = Mg\sin\theta = 0.418\,\text{kg} \times 9.81\,\text{m/s}^2 \times \sin 30° \approx 2.05\,\text{N}. \tag{33}$$

Electromagnetic braking force. By the principle of virtual work applied to the coupled electromechanical system, the generalised force conjugate to u is:

$$F_{\text{em}} = -\sum_{i=1}^{N_c} K_i(u)\, I_i(u, v). \tag{34}$$

The negative sign guarantees that F_{em} opposes the sled motion whenever K_i and I_i share the same sign, consistent with Lenz's law. Note that F_{em} is nonlinear: it depends on u through $K_i(u)$ and on v through $I_i \propto K_i v$.

Mechanical damping.

$$F_{\text{mech}} = -C_{\text{mech}}\, v, \qquad C_{\text{mech}} = C_m + C_f = 0.25 + 0.45 = 0.70\,\text{N}\cdot\text{s/m}, \tag{35}$$

where C_m is a viscous (velocity-proportional) coefficient and C_f subsumes sliding friction linearised about the operating point.

6.2 Equations of Motion

Newton's second law along the rail yields the state-space ODE:

$$\frac{du}{dt} = v, \tag{36}$$

$$\frac{dv}{dt} = \frac{1}{M}\Big(Mg\sin\theta - \sum_i K_i(u)\, I_i(u, v) - C_{\text{mech}}\, v\Big). \tag{37}$$

Energy interpretation. Multiplying Eq. (37) by Mv gives the power balance:

$$\underbrace{Mv\frac{dv}{dt}}_{\dot{E}_{\text{kin}}} = \underbrace{Mgv\sin\theta}_{\text{gravitational input}} - \underbrace{\sum_i K_i I_i v}_{P_{\text{electrical}}} - \underbrace{C_{\text{mech}} v^2}_{\text{mechanical loss}}. \tag{38}$$

This confirms that the extracted electrical power is $P_{\text{elec}} = \sum_i K_i I_i v = \sum_i V_i I_i$, consistent with Ohm's law on the coil terminals.

6.3 Sources of Nonlinearity

The system is strongly nonlinear due to three coupled mechanisms:

- **Position-dependent** $K_i(u)$: The transduction coefficient varies by up to an order of magnitude across the stroke, creating a highly non-uniform damping profile.
- **Diode activation logic**: The set of active coils changes discretely as V_L crosses individual thresholds, introducing piecewise-smooth discontinuities in F_{em}.
- **Load-dependent current redistribution**: V_L depends on N_{active} (Eq. (28)), which in turn depends on V_L.

6.4 Numerical Integration

The ODE system (Eqs. (36)–(37)) is integrated using `scipy.integrate.solve_ivp` with the LSODA method.

Initial conditions.

$$u(0) = -L/2 \quad \text{(top of rail)}, \qquad v(0) = 0.001\,\text{m/s} \quad \text{(small perturbation to start motion)}. \tag{39}$$

Terminal condition. Integration halts via event detection when:

$$u(t^*) = +L/2 \quad \Rightarrow \quad \text{simulation terminates.} \tag{40}$$

If the sled reaches the end before the time window closes, remaining frames are padded with $u = L/2$, $v = 0$.

7 Numerical Implementation

7.1 Look-Up Table for $K_i(u)$

Computing $K_i(u)$ from scratch at every ODE function evaluation would require thousands of Biot–Savart integrals per time step, making real-time integration infeasible. Instead:

1. $K_i(u)$ is pre-computed at $n_u = 60$ uniformly spaced points $u_j \in [-L/2, +L/2]$.
2. At each u_j, the flux linkage $\Phi_i(u_j)$ is computed via Eq. (21) (itself requiring $n_r = 12$ radial evaluations of the Biot–Savart integral).
3. A cubic spline $\tilde{K}_i(u)$ is fitted to the tabulated values and used for all subsequent evaluations, with extrapolation enabled for robustness at domain boundaries.

The total precomputation cost is $60 \times 10 \times 4 \times 12 = 28{,}800$ Biot–Savart integrals, each a (60×20)-point double quadrature.

7.2 Trapezoidal Quadrature Error

For the Biot–Savart integral, the trapezoidal rule converges as $\mathcal{O}(h^2)$ for smooth integrands, where $h = 2\pi/n_\theta$ in the angular direction. With $n_\theta = 60$, the angular step is $h = 0.105\,\text{rad}$ and the leading error term scales as $h^2 f''/12 \sim 10^{-3}$ of the integrand magnitude — sufficient for $\lesssim 1\%$ field accuracy.

8 Simulation Outputs and Visualization

8.1 Kinematic Panel Coordinate Transform

The kinematic side-view panel maps rail coordinates to screen pixels via two orthogonal unit vectors:

$$\hat{e}_{\text{along}} = (-\cos\theta,\ +\sin\theta), \tag{41}$$
$$\hat{e}_{\text{across}} = (-\sin\theta,\ -\cos\theta), \tag{42}$$

so that a point at (along-track distance $s_\parallel$, across-track distance $s_\perp$) maps to screen coordinates:

$$\mathbf{P}_{\text{screen}} = \mathbf{P}_0 + s_\parallel\,\hat{e}_{\text{along}} + s_\perp\,\hat{e}_{\text{across}}. \tag{43}$$

The sled centre at frame k is placed at:

$$\mathbf{P}_{\text{sled}} = \mathbf{P}_0 + \left(\frac{L}{2} + u_k\right)\sigma\,\hat{e}_{\text{along}}, \tag{44}$$

where $\sigma = 1000\,\text{mm/m}$ converts to display units. The y-axis of the panel is inverted (upper axis limit > lower) so that increasing screen-y corresponds to the physical downward direction.

8.2 Field-Line Visualisation

Magnetic field lines in the Bz panel are rendered using `matplotlib`'s `streamplot` function applied to the (B_x, B_z) vector field on the (x, z) grid. Line colour encodes field magnitude:

$$|\mathbf{B}(x, z)| = \sqrt{B_x^2 + B_z^2} \tag{45}$$

mapped to the `plasma` colourmap. The streamplot is drawn on a transparent overlay axes that is completely cleared (`cla()`) and redrawn every 4 frames; this avoids the known matplotlib issue where `FancyArrowPatch` artists accumulate and cannot be individually removed.

9 Key Parameter Summary

Symbol	Value	Unit	Description
μ_0	$4\pi \times 10^{-7}$	H/m	Permeability of free space
g	9.81	m/s^2	Gravitational acceleration
θ	30	$^\circ$	Rail inclination
L	0.40	m	Track length
B_r	1.45	T	Remanent flux density
R_m	12.5	mm	Magnet radius
d_m	25	mm	Magnet axial thickness
s_m	30	mm	Magnet pitch
N_{mag}	4	—	Number of magnets
M	418	g	Sled mass
R_{in}	8	mm	Coil inner radius
R_{out}	21	mm	Coil outer radius
d_c	12.5	mm	Coil axial depth
s_c	30	mm	Coil pitch
N_{cps}	5	—	Coils per side
N_c	10	—	Total coils
N	1200	—	Turns per coil
R_{int}	36	Ω	Coil internal resistance
R_L	10	Ω	Load resistance
V_{drop}	0.6	V	Diode forward voltage
g_{air}	1.2	mm	Air gap
z_{dist}	19.95	mm	Coil-plane offset
C_m	0.25	N s/m	Viscous damping
C_f	0.45	N s/m	Friction damping
C_{mech}	0.70	N s/m	Total mechanical damping
Δu	5×10^{-4}	m	Finite-difference step for K
δz	10^{-4}	m	Finite-difference step for B_r
n_θ	60	—	Biot–Savart angular points
n_z	20	—	Biot–Savart axial points
n_r	12	—	Flux radial integration points
n_u	60	—	Look-up table grid points
n_{frames}	300	—	Animation frames